PMM U.S.S.R.,Vol.45,pp.535-541
0021-8928/82/4 0535 $7.50/0

UDC 539.3

METHOD OF FICTITIOUS ABSORPTION IN PLANE CONTACT PROBLEMS OF THE THEORY OF ELASTICITY IN THE PRESENCE OF ADHESION*

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A method proposed for the solution of dynamical contact problems in the absence of friction /1/ is extended to the case of total adhesion. In contrast to other approaches to the solution of the problems mentioned /2—5/, this method permits efficient description of the behavior of the contact stresses in both the inner part and in the neighborhood of the edges. We note that utilization of this method in the case of a system of integral equations would require significant improvement in the approach elucidated in /1/.

1. A number of plane contact problems of the theory of elasticity on the vibration of a stamp rigidly adherent to an elastic half-space, a layer, or a layered medium, reduces to a system of integral equations in matrix form

$$\int_{-a}^{a} \mathbf{r}(x-\xi)\,\mathbf{q}(\xi)\,d\xi = \mathbf{f}(x), \quad |x| \leqslant a \tag{1.1}$$

$$\mathbf{r}(x) = \frac{1}{2\pi}\int_{\sigma} \mathbf{R}(u)\,e^{-iux}\,du, \quad \mathbf{q}(x) = \{q_1(x), q_2(x)\},$$

$$\mathbf{f}(x) = \{f_1(x), f_2(x)\}$$

Here $q_1(x)$, $q_2(x)$ are, respectively, the tangential and normal contact stresses $f_1(x), f_2(x)$ are given amplitudes of the tangential and normal displacements of points under the stamp, $2a$ is the width of the stamp. The elements $R_{mn}(u)$ $(m, n = 1, 2)$ of the matrix $\mathbf{R}(u)$ are associated with the elements of the real matrix $\mathbf{K}(u)$ by the relationships

$$R_{mm}(u) = K_{mm}(u), \quad R_{12}(u) = -R_{21}(u) = iK_{12}(u)$$

The functions $K_{mn}(u)$ are regular everywhere on the real axis with the exception of identical poles for all these functions $u = \pm p_k$ $(k = 1, 2, \ldots, n)$. The diagonal elements $K_{mm}(u)$ are even, $K_{12}(u)$ is an odd function. For $|u| \to \infty$ the functions $K_{mn}(u)$ have the following asymptotic representation

$$K_{mm}(u) = c\,|u|^{-1}[1 + O(u^{-1})], \quad c > |b|$$
$$K_{12}(u) = bu^{-1}[1 + O(u^{-1})]$$

The location of the contour σ is dictated by the principle of ultimate absorption and is determined in conformity with the rules set up in /6/.

For the properties of the kernels mentioned, the system of integral equations (1.1) is uniquely solvable in $L_\alpha(-a, a), \alpha > 1$. Uniqueness criteria were determined in /6,7/.

We represent $\mathbf{R}(u)$ as the product of two matrices

$$\mathbf{R}(u) = \mathbf{S}(u)\,\mathbf{\Pi}(u)$$

so that the behavior of the elements s_{mn} of the matrix $\mathbf{S}(u)$ at infinity would agree with the behavior of the corresponding elements r_{mn} of the matrix $\mathbf{R}(u)$. Evidently, the matrix $\boldsymbol{\pi}(u)$ with real elements, associated with $\mathbf{\Pi}(u)$ by the relationships

$$\Pi_{mm} = \pi_{mm}, \quad \Pi_{12} = i\pi_{12}, \quad \Pi_{21} = -i\pi_{21} \tag{1.2}$$

will then possess the property

$$\boldsymbol{\pi}(u) = \mathbf{I} + \mathbf{O}(u^{-k}), \quad k > 0, \quad |u| \to \infty \tag{1.3}$$

Here the matrix $\mathbf{I}$ is the unit, while the matrix $\mathbf{O}$ has elements that decrease as a power at infinity.

*Prikl.Matem.Mekhan.,45,No.4,725-733,1981

We select $s_{mn}(u)$ in the following form

$$s_{11} = s_{22} = \beta^2 (u^2 + B^2)^{-1/2} \operatorname{ch} 2\alpha\varphi \qquad (1.4)$$
$$s_{12} = -s_{21} = i\beta^2 (u^2 + B^2)^{-1/2} \operatorname{sh} 2\alpha\varphi$$
$$\varphi = \operatorname{arctg} \frac{u}{B}, \quad \alpha = \pi^{-1} \operatorname{arcth} \frac{b}{c}, \quad \beta^4 = c^2 - b^2$$

B is a parameter whose selection will be discussed below.

To construct the approximate solution, the matrix $\pi(u)$ (and therefore $\Pi(u)$ as well) is approximated by the matrix $\pi_*(u)$ with rational elements from the condition of nearness of its elements in absolute value to the corresponding elements of $\pi(u)$ with a given degree of accuracy. According to the theorem set up in /7/, this assures the nearness of the solutions of systems of equations with kernels described by the matrices $\pi(u)$ and $\pi_*(u)$.

As a result of approximating the elements of the matrix $\pi(u)$ by rational functions by using Bernshtein polynomials, they can be represented as

$$\pi_{mm} = \prod_{k=1}^{n} (u^2 - z_{kmm}^2)(u^2 - p_k^2)^{-1} \qquad (1.5)$$
$$\pi_{mn} = u \prod_{k=1}^{n-1} (u^2 - z_{kmn}^2) \prod_{k=1}^{n} (u^2 - p_k^2)^{-1}, \quad m \neq n$$

Here z_{kmn} and p_k are, respectively, zeroes and poles of elements of the matrix $\pi(u)$ lying above the contour σ.

After the approximation, the elements π_{mn} evidently possess the property (1.3) as before. The approximation of the inverse matrix is constructed analogously. Let us note that elements of the inverse matrix Π^{-1} are related to the elements of the real matrix $\pi^{-1}(u)$ by

$$\Pi_{mm}^{-1} = \pi_{mm}^{-1}, \quad \Pi_{12}^{-1} = -i\pi_{12}^{-1}, \quad \Pi_{21}^{-1} = i\pi_{21}^{-1} \qquad (1.6)$$

We later need the following representations of the matrices $\pi(u)$ and $\pi^{-1}(u)$:

$$\pi(u) = \mathbf{I} + \mathbf{h}(u), \quad \pi^{-1}(u) = \mathbf{I} + \mathbf{p}(u) \qquad (1.7)$$

The elements of the matrices $\mathbf{h}(u)$ and $\mathbf{p}(u)$ have the form

$$h_{mm} = \sum_{i=1}^{n} a_{mm}^{i} (u^2 - p_i^2)^{-1}, \quad h_{mn} = u \sum_{i=1}^{n} (u^2 - p_i^2)^{-1} a_{mn}^{i}, \quad m \neq n$$
$$a_{mm}^{i} = \prod_{k=1}^{n} (p_i^2 - z_{kmm}^2) \prod_{\substack{k=1 \\ k \neq i}}^{n} (p_i^2 - p_k^2)^{-1}$$
$$a_{mn}^{i} = \prod_{k=1}^{n-1} (p_i^2 - z_{kmn}^2) \prod_{\substack{k=1 \\ k \neq i}}^{n} (p_i^2 - p_k^2)^{-1}, \quad m \neq n$$
$$p_{mm} = \prod_{j=1}^{n} \beta_{mm}^{j} (u^2 - \zeta_j^2)^{-1}; \quad p_{mn} = u \prod_{j=1}^{n} \beta_{mn}^{j} (u^2 - \zeta_j^2)^{-1}, \quad m \neq n$$
$$\beta_{mm}^{j} = \prod_{k=1}^{n} (\zeta_j^2 - \gamma_{kmm}^2) \prod_{\substack{k=1 \\ k \neq j}}^{n} (\zeta_j^2 - \zeta_k^2)^{-1}$$
$$\beta_{mn}^{j} = \prod_{k=1}^{n-1} (\zeta_j^2 - \gamma_{kmn}^2) \prod_{\substack{k=1 \\ k \neq j}}^{n} (\zeta_j^2 - \zeta_k^2)^{-1}, \quad m \neq n$$

Here γ_{kmn} and ζ_k are, respectively, zeroes and poles of elements of the inverse matrix lying above the contour σ, where all the elements π_{mn}^{-1} have the same pole.

2. Applying the method from /8/, we seek the solution of the matrix integral equation (1.1) in $L_\alpha(-a, a)$, $\alpha > 1$ in the form

$$\mathbf{q}(x) = \mathbf{q}_0(x) + \boldsymbol{\varphi}(x) \qquad (2.1)$$

so that the functionals

$$\int_{-a}^{a} \mathbf{q}(x) e^{\pm ip_k x} dx = \int_{-a}^{a} \boldsymbol{\varphi}(x) e^{\pm ip_k x} dx, \quad k = 1, 2, \ldots, n$$

would agree (the p_k have been determined earlier). This same functional of the inknown vector function $\mathbf{q}_0(x)$ equals zero.

We take a system of delta functions with carriers at the points $x_k = \pm y_k$ as components of the vector $\boldsymbol{\varphi}(x)$, where y_k are points separating the interval $(0, a)$ into equal segments

$$\boldsymbol{\varphi}(x) = \sum_{k=1}^{2n} \mathbf{c}_k \delta(x - x_k), \quad \mathbf{c}_k = \{c_{1k}, c_{2k}\} \tag{2.2}$$

(c_{1k}, c_{2k} are constants to be determined).

We introduce the new unknown vector function $\mathbf{t}(x)$ by the relationship

$$\mathbf{t}(x) = \frac{1}{2\pi} \int_{\sigma} \mathbf{T}(u) e^{-iux} du \tag{2.3}$$

$$\mathbf{T}(u) = \boldsymbol{\Pi}(u) \mathbf{Q}_0(u), \quad \mathbf{Q}_0(u) = \int_{-a}^{a} \mathbf{q}_0(x) e^{iux} dx$$

Inserting (2.1), (2.2) into (1.1), we arrive at the following integral equation

$$\int_{-a}^{a} \mathbf{k}_0(x - \xi) \mathbf{t}(\xi) d\xi = \mathbf{f}(x) - \sum_{k=1}^{2n} \mathbf{c}_k \mathbf{k}(x - x_k) \tag{2.4}$$

$$\mathbf{k}_0(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathbf{S}(u) e^{-iux} du$$

The elements s_{mn} of the matrix $\mathbf{S}(u)$ are given by (1.4).

Therefore, the system of integral equations of the dynamical contact problem (1.1) was reduced to a matrix integral equation of the static contact problem with matrix functions $\mathbf{S}(u)$ whose elements have no singularities on the real axis.

Without spoiling the generality we set

$$\mathbf{f}(x) = \{A_1 e^{-i\eta_1 x}, A_2 e^{-i\eta_2 x}\} \tag{2.5}$$

Let $\mathbf{t}_0(x)$ be the solution of (2.4) with the right side (2.5), and $\mathbf{t}_0^*(x)$ the solution corresponding to the right side

$$\mathbf{f}(x) = \{c_{1k} e^{-i\eta x}, c_{2k} e^{-i\eta x}\}$$

Then the solution $\mathbf{t}(x)$ of the integral equation (2.4) and the Fourier transform of this solution will be obtained in the form

$$\mathbf{t}(x) = \mathbf{t}_0(x) - \boldsymbol{\varphi}(x) - \frac{1}{4\pi^2} \sum_{k=1}^{2n} \int_{\sigma} \mathbf{L}(u, x_k) e^{-iux} du \tag{2.6}$$

$$\mathbf{T}(u) = \mathbf{T}_0(u) - \sum_{k=1}^{2n} \mathbf{c}_k e^{iux_k} - \frac{1}{2\pi} \sum_{k=1}^{2n} \mathbf{L}(u, x_k) \tag{2.7}$$

Here

$$\mathbf{L}(u, x_k) = \int_{\sigma} \mathbf{M}(u, \eta) \mathbf{T}_0^*(u) e^{i\eta x_k} d\eta, \quad \mathbf{L}(u, x_k) = \{L_1, L_2\} \tag{2.8}$$

$$\mathbf{M}(u, \eta) = \mathbf{S}^{-1}(u) \mathbf{S}(\eta) [\boldsymbol{\Pi}(\eta) - \mathbf{I}] \mathbf{S}(u)$$

$\mathbf{S}^{-1}(u)$ is the matrix inverse to $\mathbf{S}(u), \mathbf{T}_0^*(u), \mathbf{T}_0(u), \mathbf{T}(u)$ are Fourier transforms of the vector functions $\mathbf{t}_0^*(x), \mathbf{t}_0(x)$ and $\mathbf{t}(x)$, respectively.

We find the unknown vector $\mathbf{c}_k$ by using a lemma from /8/ according to which the following identity holds

$$T(\pm \zeta_k) \equiv 0, \quad k = 1, 2, \ldots, n \tag{2.9}$$

where $\mathbf{T}(u)$ is given by (2.7), and ζ_k were determined in Sect.1. The relationships (2.9) are an algebraic system of order $4n$ to determine the $4n$ constants c_k.

We determine the unknown vector-function $\mathbf{q}_0(x)$ from (2.3)

$$\mathbf{q}_0(x) = \mathbf{t}(x) - \frac{1}{2\pi}\int_\sigma [\mathbf{\Pi}^{-1}(u) - \mathbf{I}]\,\mathbf{T}(u)\,e^{-iux}du \qquad (2.10)$$

and we obtain the solution of (1.1) by using (2.1), (2.2) and (2.10), as

$$\mathbf{q}(x) = \mathbf{t}_0(x) - \frac{1}{2\pi}\int_\sigma [\mathbf{\Pi}^{-1}(u) - \mathbf{I}]\,\mathbf{T}_0(u)\,e^{-iux}du \qquad (2.11)$$

$$\frac{1}{4\pi^2}\sum_{k=1}^{2n}\int_\sigma \mathbf{\Pi}^{-1}(u)\,\mathbf{L}(u, x_k)\,e^{-iux}du - \frac{1}{2\pi}\sum_{k=1}^{2n}\int_\sigma [\mathbf{\Pi}^{-1}(u) - \mathbf{I}]\,\mathbf{c}_k e^{-iu(x-x_k)}du$$

3. The solution $\mathbf{t}_0(x)$ can be constructed by the factorization method /4/. In this case, the matrix function $\mathbf{S}(u)$ must be factored, i.e., represented in the form

$$\mathbf{S}(u) = \mathbf{C}_-(u)\,\mathbf{D}_+(u) = \mathbf{M}_+(u)\,\mathbf{N}_-(u) \qquad (3.1)$$

The matrices $\mathbf{D}_+(u)$ and $\mathbf{M}_+(u)$ have elements and determinants that are regular above the contour σ, are continuous on it, where the determinants have no zeroes in this domain. Similarly for $\mathbf{C}_-(u)$, $\mathbf{N}_-(u)$ in the domain below the contour σ. The elements of all the matrices in the domains of regularity decrease at infinity.

Factoring of the $\mathbf{S}(u)$ from (3.1) is accomplished explicitly in conformity with the general theorems in /9/. The contour σ agrees in this case with the real axis. The elements of the matrices $\mathbf{C}_-(u)$, $\mathbf{D}_+(u)$ $\mathbf{M}_+(u)$, $\mathbf{N}_-(u)$ have the form

$$\begin{aligned} d_{11} &= -id_{12} = -2im_{12}\beta^{-2} = 2m_{22}\beta^{-2} = (B - iu)^{-\nu_1} \\ id_{21} &= -d_{22} = 2m_{11}\beta^{-2} = -2im_{21}\beta^{-2} = (B - iu)^{-\nu_1} \\ n_{11} &= in_{12} = -2ic_{12}\beta^{-2} = -2c_{22}\beta^{-2} = (B + iu)^{-\nu_1} \\ in_{21} &= n_{22} = 2c_{11}\beta^{-2} = 2ic_{21}\beta^{-2} = (B + iu)^{-\nu_1} \end{aligned} \qquad (3.2)$$

The components $T_{0i}(u)$ of the vector $\mathbf{T}_0(u)$ are obtained by applying the above method, in the form

$$T_{0k}(u) = 2\pi\sum_{s=1}^{2} A_s S_{ks}(u)\,\delta(u - \eta_s) - i\sum_{s=1}^{2}\sum_{j=1}^{2}\frac{A_j}{u - \eta_j}\,[N_{ks}(u)\,M_{sj}(\eta_j)\,e^{i\alpha(u-\eta_j)} - D_{ks}(u)\,C_{sj}(\eta_j)\,e^{-i\alpha(u-\eta_j)}], \quad k = 1, 2 \qquad (3.3)$$

We note that S_{ks}, D_{ks}, C_{sj}, N_{ks}, M_{sj} are elements of the inverse matrices $\mathbf{S}^{-1}(u)$, $\mathbf{D}_+^{-1}(u)$, $\mathbf{C}_-^{-1}(u)$, $\mathbf{N}_-^{-1}(u)$, $\mathbf{M}_+^{-1}(u)$. Elements of the direct matrices are given by (1.4) and (3.2).

Applying the inverse Fourier transform to (3.3), we find $\mathbf{t}_0(x) = \{t_{01}, t_{02}\}$ in the form

$$\begin{aligned} t_{01}(x) &= -A_1\beta^{-2}\sqrt{B^2 - \eta_1^2}\,e^{-i\eta_1 x}\,[e^{2\alpha\varphi_1}G(\eta_1, x) + \\ &\quad e^{-2\alpha\varphi_1}G(-\eta_1, -x) - \operatorname{ch} 2\alpha\varphi_1] + iA_2\beta^{-2}\sqrt{B^2 + \eta_2^2}\,e^{-i\eta_2 x} \times \\ &\quad [e^{2\alpha\varphi_2}G(\eta_2, x) - e^{-2\alpha\varphi_2}G(-\eta_2, -x) - \operatorname{sh} 2\alpha\varphi_2] \\ t_{02}(x) &= -iA_1\beta^{-2}\sqrt{B^2 + \eta_1^2}\,e^{-i\eta_1 x}\,[e^{2\alpha\varphi_1}G(\eta_1, x) - \\ &\quad e^{-2\alpha\varphi_1}G(-\eta_1, -x) - \operatorname{sh} 2\alpha\varphi_1] - A_2\beta^{-2}\sqrt{B^2 + \eta_2^2}\,e^{-i\eta_2 x} \times \\ &\quad [e^{2\alpha\varphi_2}G(\eta_2, x) + e^{-2\alpha\varphi_2}G(-\eta_2, -x) - \operatorname{ch} 2\alpha\varphi_2] \\ G(\eta, x) &= \frac{1}{2}\,[g(\alpha, \eta, x) - g(-\alpha, -\eta, -x)] \\ g(\alpha, \eta, x) &= \Gamma^{-1}(\nu_2)\,\Gamma\,[\nu_2, (B + i\eta)(a - x)] \\ \varphi_{1,2} &= \operatorname{arctg}\frac{\eta_{1,2}}{B}, \quad \nu_{1,2} = -\frac{1}{2} \mp i\alpha \end{aligned} \qquad (3.4)$$

($\Gamma(\alpha, x)$ is the incomplete Gamma function, and $\Gamma(x)$ is the Gamma function).

The vector functions $\mathbf{t}_0^*(x)$, $\mathbf{T}_0^*(u)$ are determined from (3.3) and (3.4) if we put $\eta_1 = \eta_2 = \eta$, $A_1 = c_{1k}$, $A_2 = c_{2k}$.

4. We obtain the final formulas to compute the tangential and normal stresses under the stamp. We insert the expressions for $\mathbf{t}_0(x)$, $\mathbf{T}_0(u)$, $\mathbf{T}_0^*(u)$, found in Sect.3, in the integral representation of the solution (2.11) and we use the approximation (1.5) and the relations (1.7). The integrals L_i $(i = 1, 2)$, defined by (2.8), are calculated by residues after having been multiplied by the matrices and a number of transformations since the integrands decrease exponentially in the lower half-plane of the complex variable η and have no branch points. The remaining integrals in the solution (2.11) are evaluated by operational calculus formulas.

Omitting the computations, we present the general form of the approximate solution of the system of equations (1.1).

$$q_1(x) = G_1(x) - \frac{i}{2}\sum_{k=1}^{2n}[E_{11}(x, x_k)c_{1k} + iE_{12}(x, x_k)c_{2k}] \tag{4.1}$$

$$q_2(x) = G_2(x) - \frac{i}{2}\sum_{k=1}^{2n}[-iE_{21}(x, x_k)c_{1k} + E_{22}(x, x_k)c_{2k}]$$

The functions $G_1(x)$, $G_2(x)$, $E_{mn}(x, x_k)$ $(m, n = 1, 2)$ have the form

$$\begin{aligned}G_1(x) = &-A_1e^{-i\eta_1 x}K_{11}^{-1}(\eta_1) + iA_2e^{-i\eta_2 x}K_{12}^{-1}(\eta_2) + \\ &A_1\beta^{-2}\sqrt{B^2+\eta_1^2}\,e^{-i\eta_1 x}[F^+(\eta_1, x)\Pi_{11}^{-1}(\eta_1) + F^-(\eta_1, x)\Pi_{12}^{-1}(\eta_1)] - \\ &iA_2\beta^{-2}\sqrt{B^2+\eta_2^2}\,e^{-i\eta_2 x}[F^-(\eta_2, x)\Pi_{11}^{-1}(\eta_2) + F^+(\eta_2, x)\Pi_{12}^{-1}(\eta_2)] - \\ &A_1(2\beta)^{-2}\sum_{j=1}^{n}[\beta_{11}^j\zeta_j^{-1}X_{1j}^-(\eta_1, x) - \beta_{12}^jX_{2j}^+(\eta_1, x)] + \\ &iA_2(2\beta)^{-2}\sum_{j=1}^{n}[\beta_{11}^j\zeta_j^{-1}Y_{2j}^-(\eta_2, x) - \beta_{12}^jY_{1j}^+(\eta_2, x)] + \\ &e^{-B(a-x)}\frac{1}{2}\beta^{-2}\{(a-x)^{\nu_2}[A_1\Psi(\alpha, \eta_1) - iA_2\Psi(\alpha, \eta_2)] + \\ &(a-x)^{\nu_1}[A_1\Psi(-\alpha, \eta_1) + iA_2\Psi(-\alpha, \eta_2)]\} + \\ &e^{-B(a+x)}\frac{1}{2}\beta^{-2}\{(a+x)^{\nu_2}[A_1\Psi(\alpha, -\eta_1) + iA_2\Psi(\alpha, -\eta_2)] + \\ &(a+x)^{\nu_1}[A_1\Psi(-\alpha, -\eta_1) - iA_2\Psi(-\alpha, -\eta_2)]\}\end{aligned}$$

We obtain the function $G_2(x)$ if we replace

$$A_1, A_2, \eta_1, \eta_2, K_{11}^{-1}, \beta_{11}^j, \beta_{12}^j \quad \text{by} \quad A_2, -A_1, \eta_2, \eta_1, K_{22}^{-1}, \beta_{22}^j, \beta_{21}^j$$

respectively, in $G_1(x)$

The functions $E_{mn}(x, x_k)$ are determined by the expressions

$$\begin{aligned}E_{11}(x, x_k) = &D_1^+(\alpha, x, x_k) + D_1^-(-\alpha, x, x_k) + D_1^+(\alpha, -x, -x_k) + \\ &D_1^-(-\alpha, -x, -x_k) - \frac{1}{2}\sum_{j=1}^{n}\{\beta_{11}^j\zeta_j^{-1}[B_{1j}^-(x, x_k) + \\ &B_{1j}^-(-x, -x_k)] - \beta_{12}^j[\varepsilon_{2j}^+(x, x_k) + \varepsilon_{2j}^+(-x, -x_k)]\}\end{aligned}$$

$$\begin{aligned}E_{12}(x, x_k) = &-D_1^+(\alpha, x, x_k) + D_1^-(-\alpha, x, x_k) + D_1^+(\alpha, -x, -x_k) - \\ &D_1^-(-\alpha, -x, -x_k) + \frac{1}{2}\sum_{j=1}^{n}\{\beta_{11}^j\zeta_j^{-1}[\varepsilon_{1j}^-(x, x_k) - \varepsilon_{1j}^-(-x, -x_k)] - \\ &\beta_{12}^j[B_{2j}^+(x, x_k) - B_{2j}^+(-x, -x_k)]\}\end{aligned}$$

We obtain the functions $E_{22}(x, x_k)$, $E_{21}(x, x_k)$ by, respectively, replacing in

$$E_{11}(x, x_k),\ E_{12}(x, x_k)\ \beta_{11}^j, \beta_{12}^j, D_1^{\pm}, \varepsilon_{1j}^-, B_{2j}^+$$
$$\varepsilon_{2j}^+, B_{1j}^- \quad \text{by} \quad \beta_{22}^j, \beta_{21}^j, D_2^{\pm}, \varepsilon_{2j}^-, B_{1j}^+, \varepsilon_{1j}^+, B_{2j}^-.$$

The following notation was used:

$$\Psi(\alpha, \eta) = \Gamma^{-1}(\nu_2 + 1)(B - i\eta)^{-\nu_1}e^{-i\eta a}, \quad \nu_{1,2} = -1/2 \mp i\alpha$$
$$F^{\pm}(\eta, x) = e^{2\alpha\varphi_1}F(\eta, x) \pm e^{-2\alpha\varphi_1}F(-\eta, -x)$$
$$F(\eta, x) = \frac{1}{2}[f(\alpha, \eta, x) + f(-\alpha, -\eta, -x)]$$

$$f(\alpha,\ \eta,\ x) = \Gamma^{-1}(\nu_2+1)\,\gamma\,[\nu_2+1,\ (B+i\eta)(a-x)]$$
$$X_{kj}^{\pm}(\eta,\ x) = e^{-ia\eta}\chi_{kj}^{\pm}(\eta,\ x) + e^{ia\eta}\chi_{kj}^{\pm}(-\eta,\ -x)$$
$$Y_{kj}^{\pm}(\eta,\ x) = e^{-ia\eta}\chi_{kj}^{\pm}(\eta,\ x) - e^{ia\eta}\chi_{kj}^{\pm}(-\eta,\ -x)$$
$$\chi_{1j}^{\pm}(\eta,\ x) = (B-i\eta)^{-\nu_1}\Phi_j^{\pm}(\alpha,\ \eta,\ x) + (B-i\eta)^{-\nu_1}\Phi_j^{\pm}(-\alpha,\ \eta,\ x)$$
$$\chi_{2j}^{\pm}(\eta,\ x) = (B-i\eta)^{-\nu_1}\Phi_j^{\pm}(\alpha,\ \eta,\ x) - (B-i\eta)^{-\nu_1}\Phi_{j\pm}(-\alpha,\ \eta,\ x)$$
$$\Phi_j^{\pm}(\alpha,\eta,x) = \frac{(B-i\zeta_j)^{-\nu_2}}{\zeta_j+\eta}e^{-i\zeta_j(a-x)}\,[1-f(\alpha,-\zeta_j,x)] \pm \frac{(B+i\zeta_j)^{-\nu_2}}{\zeta_j-\eta}e^{i\zeta_j(a-x)}f(\alpha,\zeta_j,x)$$
$$D_m^{\pm}(\alpha,x,x_k) = (a-x)^{\nu_2}\Gamma^{-1}(\nu_2+1)\,e^{-B(a-x)}\sum_{i=1}^{n}\alpha_{mi}^{\pm}(B-ip_i)^{\nu_2}\times e^{ip_i(a-x_k)},\quad m=1,2$$
$$B_{mj}^{\pm}(x,x_k) = \sum_{i=1}^{n}[\alpha_{mi}^{+}(B-ip_i)^{\nu_2}\Phi_j^{\pm}(\alpha,-p_i,x) + \alpha_{mi}^{-}(B-ip_i)^{\nu_1}\Phi_j^{\pm}(-\alpha,-p_i,x)]\,e^{ip_i(a-x_k)},\quad m=1,2$$
$$\varepsilon_{mj}^{\pm}(x,x_k) = \sum_{i=1}^{n}[\alpha_{mi}^{+}(B-ip_i)^{\nu_2}\Phi_j^{\pm}(\alpha,-p_i,x) - \alpha_{mi}^{-}(B-ip_i)^{\nu_1}\Phi_j^{\pm}(-\alpha,-p_i,x)]\,e^{ip_i(a-x_k)},\quad m=1,2$$
$$\alpha_{1i}^{\pm} = \frac{\alpha_{11}^{i}\pm\alpha_{12}^{i}}{2p_i},\quad \alpha_{2i}^{\pm} = \frac{\alpha_{22}^{i}\pm\alpha_{21}^{i}}{2p_i}$$

($\gamma(\alpha,\ x)$ is the incomplete Gamma function).

A packet of programs was compiled by the formulas (4.1) presented above by using the BESM-6 to compute the contact stresses as well as to determine the unknown vector c_k from the condition (2.9).

5. As an illustration, we consider the plane problem of harmonic vibration of a stamp of width $2a$ on an elastic layer of thickness $2h$ connected rigidly to an undeformable base. The stamp adheres rigidly to the layer. The problem is reduced to an equation of the form (1.1). The elements of the matrix $\mathbf{K}(u)$ in this case have the form

$$K_{11}(u) = {}^1/_2\varkappa_2^2(\sigma_2\,\mathrm{sh}\,2\sigma_2\,\mathrm{ch}\,2\sigma_1 - \sigma_1^{-1}u^2\,\mathrm{sh}\,2\sigma_1\,\mathrm{ch}\,2\sigma_2)\Delta^{-1}(u)$$
$$K_{22}(u) = {}^1/_2\varkappa_2^2(\sigma_1\,\mathrm{sh}\,2\sigma_1\,\mathrm{ch}\,2\sigma_2 - \sigma_2^{-1}u^2\,\mathrm{sh}\,2\sigma_2\,\mathrm{ch}\,2\sigma_1)\Delta^{-1}(u)$$
$$K_{12}(u) = -u\,\{(2u^2 - {}^1/_2\varkappa_2^2)(1-\mathrm{ch}\,2\sigma_1\,\mathrm{ch}\,2\sigma_2) + \sigma_1^{-1}\sigma_2^{-1}[2u^4 - u^2({}^3/_2\varkappa_2^2+\varkappa_1^2)+\varkappa_1^2\varkappa_2^2]\,\mathrm{sh}\,2\sigma_1\,\mathrm{sh}\,2\sigma_2\}\,\Delta^{-1}(u)$$
$$\Delta(u) = u^2(2u^2-\varkappa_2^2) - (2u^4 - u^2\varkappa_2^2 + {}^1/_4\varkappa_2^4)\,\mathrm{ch}\,2\sigma_1\,\mathrm{ch}\,2\sigma_2 + \sigma_1^{-1}\sigma_2^{-1}u^2[2u^4 - u^2(2\varkappa_2^2+\varkappa_1^2) + \varkappa_1^2\varkappa_2^2 + {}^1/_4\varkappa_2^4]\,\mathrm{sh}\,2\sigma_1\,\mathrm{sh}\,2\sigma_2$$
$$\varkappa_1^2 = \omega^2\rho h^2(\lambda+2\mu)^{-1},\quad \varkappa_2^2 = \omega^2\rho h^2\mu^{-1},\quad \sigma_k = \sqrt{u^2-\varkappa_k^2},\quad k=1,2$$

The quantities $f_i(x)$ are taken with the factor $2\mu h^{-1}$, a is the dimensionless half-width of the stamp ν,ρ are the Poisson's ratio and density of the material, and λ,μ are the Lamé coefficients.

The elements of the matrix $\mathbf{K}(u)$ possess the properties listed above. The matrix functions

$$\Pi(u) = \mathbf{S}^{-1}(u)\,\mathbf{R}(u),$$
$$\Pi^{-1}(u) = \mathbf{R}^{-1}(u)\,\mathbf{S}(u)$$

are constructed, whose elements are associated with the real elements of the matrices $\pi(u)$ and $\pi^{-1}(u)$ by the relations (1.2) and (1.6).

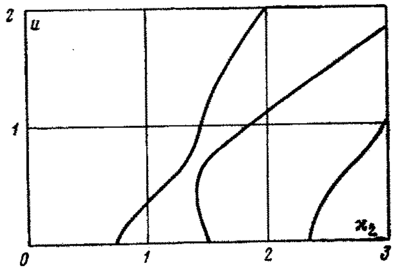

Fig.1

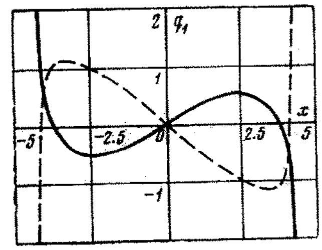

Fig.2

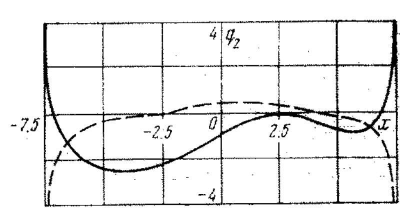

Fig.3

To construct the approximating matrices, the neutral curves (curves of zero and poles) of the elements of the matrices $\pi(u)$ and $\pi^{-1}(u)$ are constructed first. An example of the distribution of the real poles of the elements of the matrix $\pi(u)$ for $\nu = 0.3$ is presented in Fig.1. By using Bernshtein polynomials, the functions $\pi_{mn}(u)$ are simultaneously approximated by rational functions of the form (1.5) with any previously assigned accuracy. The approximation of the inverse matrix $\pi^{-1}(u)$ is analogous. The matrix $\pi^{-1}(u)$ can be obtained directly from $\pi(u)$ but this path will result in an increase in the number of coefficients c_{1k}, c_{2k}, and therefore, the order of the system to determine these constants doubles. To construct effective approximate solutions of the system of integral equations (1.1), the parameter B should be taken as large as possible. Let us note that this results in an increase in the order of the approximating polynomials for a given accuracy of the approximation. For numerical computations $B = 10$ was taken.

A computation of the contact stresses was performed on a BESM-6 by using (4.1) as a function of the reduced frequency $\varkappa_2$, the Poisson's ratio ν, and the stamp width $2a$.

Graphs of the complex tangential stresses $q_1(x)$ are presented in Fig.2 for $\varkappa_2 = 1.8$, $a = 5$, $A_1 = 0$, $A_2 = 1$, $\nu = 0.3$, $\eta_1 = \eta_2 = 0$ and normal stresses $q_2(x)$ for $\varkappa_2 = 1.8$, $a = 7.5$, $A_1 = A_2 = 1$, $\nu = 0.3$, $\eta_1 = \eta_2 = 0$ in Fig.3. Graphs of the real part are represented by solid, and of the imaginary parts, by dashed lines.

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Translated by M.D.F.